

# A Pareto-based Symbiotic Relationships Model for Unconstrained Continuous Optimization

Leanderson André

Graduate Program in Applied Computing  
State University of Santa Catarina  
Joinville-SC, Brazil  
Email: leanderson.andre@gmail.com

Rafael Stubs Parpinelli

Graduate Program in Applied Computing  
State University of Santa Catarina  
Joinville-SC, Brazil  
Email: rafael.parpinelli@udesc.br

**Abstract**—Symbiotic relationships are one of several phenomena that can be observed in nature. These relationships consist of interactions between organisms and can lead to benefits or damages to those involved. In an optimization context, symbiotic relationships can be used to perform information exchange between populations of candidate solutions to a given problem. This paper presents an information exchange model inspired by symbiotic relationships and applies the model to unconstrained single-objective continuous optimization problems. The symbiotic relationships are modelled using the Pareto dominance criteria inside a computational ecosystem for optimization. The Artificial Bee Colony algorithm is used to compound the populations of the ecosystem. Four models of relationships are analyzed: slavery, competition, altruism and mutualism. Thirty unconstrained single-objective continuous benchmark functions with high number of dimensions ( $d = 200$ ) are tested and obtained results compared. Results suggest that the proposed model for information exchange favors the balance between exploration and exploitation leading to better results.

## I. INTRODUCTION

From observations and understanding of natural phenomena, several technologies have been developed for different applications. For example, the sonar was developed inspired by the echolocation of bats and dolphins, airplanes inspired by birds, among others [1]. In the same way, Nature has inspired the development of biologically plausible algorithms. The main feature of bio-plausible systems is the use of natural inspirations at some degree where the designers of these systems generally aim to achieve biologically plausible functionalities in non-biological contexts, such as the optimization of engineering problems [2]. These algorithms are recognized and are part of a research field known as Natural Computing [1][3].

Symbiotic relationships are one of several phenomena that can be observed in nature. These relationships are carried by interactions that individuals perform with each other in an ecosystem and are diverse as mutualism, amensalism, predatism, society, slavery, among others [4].

Symbiotic relationships play an important role in the ecosystem biological control (e.g., diversity, extinction, food chain). Moreover, an organism can not live alone. This means that, directly or indirectly, each organism depends on another to feed, reproduce or survive. Another important point is that symbiotic relationships are responsible for the coevolution of

species. For example, predators evolve their tactics to capture preys, and preys evolve their tactics to avoid predators. This is a phenomenon called arms-race [4].

Modeling symbiotic relationships as part of optimization algorithms increases their biological plausibility, i.e. makes them more similar to biological systems. Moreover, the inclusion of symbiotic relationships can improve the coevolution of populations of candidate solutions as well as influencing their diversities. The main hypothesis that we investigate in this work is if a system with more biologically plausible features is able to enhance its performance. The inspiration is that biological systems are able to handle complex problems and tasks naturally [2]. It is important to emphasize that a particular feature of an optimization algorithm is biologically plausible when it is similar to the biological phenomenon in which it is inspired.

The use of interactions between populations and information exchange are not new in the field of optimization. An example is the well-known island model GA [5] and other algorithms that apply the same concept (e.g., PSO [6] and ACO [7]). In the present work the topologies are not static and do not follow a standard formation like ring, star or fully connected as performed by the island model. The topologies are dynamic, i.e., topologies can assume different patterns [8].

The concept of symbiotic relationships has already been applied as optimization routines. In [9] it is proposed an optimization algorithm inspired by relationships of commensalism, predatism and mutualism to solve continuous functions and in [10] a host-parasite model is applied to solve big deceptive problems. This work presents a different information exchange model inspired by symbiotic relationships that is based on the Pareto dominance criteria.

The model is applied in an ecologically-inspired approach for optimization called ECO[11][12]. The well known Artificial Bee Colony algorithm [13] is used to compound the populations of the ecosystem [8]. The ecologically-based algorithm is a co-evolutionary framework and employs multiple populations for unconstrained single-objective continuous optimization. Thus, the proposed model defines different ways to interact these populations. Interactions are conducted through the exchange of information between two distinct populations. Both the number of individuals and which individuals are

selected in these interactions are defined probabilistically. This feature is the main difference from the original model described in [11][12].

This paper is structured as follows. Section II provides an overview of the standard ecologically-inspired algorithm. Section III presents the computational model of symbiotic relationships. The experiments and results are presented in Section IV. Finally, Section V concludes the paper with final remarks and future research.

## II. THE COMPUTATIONAL ECOSYSTEM FOR OPTIMIZATION

The ecologically-inspired algorithm, named ECO, represents a perspective to apply optimization strategies cooperatively in an ecosystemic context [11]. ECO is composed by populations of individuals ( $Q$ ) and each population evolves according to an optimization strategy.

The ecological inspiration stems from the use of some ecological concepts, such as habitats, ecological relationships and ecological successions [4]. A habitat is the actual location in the environment where an organism lives and consists of all the physical and biological resources available. In this way, populations of individuals that are scattered in the search surface and established in the same region constitute an ecological habitat. The ecosystem can be depicted in three levels. The lower level represents the problem-dependent search space that defines a hypersurface and, as well as in nature, populations can move around through all of it. The movement of populations can be observed by changing the values of variables that affect function  $f(\cdot)$ . The system is composed by several habitats that can also interact to each other.

With the definition of habitats, two categories of ecological relationships can be defined. Intra-habitats relationships that occur between populations inside each habitat, and inter-habitats relationships that occur between habitats [4].

In ECO, the mating relationship represents the intra-habitats information exchange. Populations belonging to the same habitat can establish a reproductive link between their individuals, favoring the co-evolution of the involved populations through competition for mating. Populations belonging to different habitats are reproductively isolated. The intra-habitats communication topologies represents the intermediate level of ECO.

Great migrations represents inter-habitats relationships. Individuals belonging to a given habitat can migrate to other habitats aiming at identifying promising areas for survival and mating. The inter-habitats communication topologies represents the upper level of the system.

Inside the ecological metaphor, ecological successions represent the transformational process of the system. In this process, population groups are formed (habitats), relationships between populations are established and the system stabilizes by means of the self-organization of its components.

It is important to highlight that the concept of interactions between populations is not new. An example is the well-known island model GA [5] and other algorithms that apply the same

concept (e.g., PSO [6] and ACO [7]). However, the approach used in ECO differs from the others by presenting a new level of abstraction for the topologies of communication. There are two different topologies of communication, being the intra and inter-habitats communications. The formation of topologies is done probabilistically and is influenced by the distribution of populations on the surface of function  $f(\cdot)$ . It can also be observed that the topologies are not static and do not follow a standard formation like ring, star or fully connected as performed by the island model. The topologies are dynamic, i.e., topologies can assume different patterns [8] at every given moment  $t$ .

Another important feature is that ECO enables the use of any optimization algorithm to evolve the populations cooperatively. Each population can behave according to the mechanisms of intensification and diversification, tuned by the control parameters, specific of an optimization strategy.

Algorithm 1 shows the pseudo-code of ECO. The ecological succession loop (lines 3 to 12) refers to iterations of the computational ecosystem. In line 4, evolutive period, each population evolves (generations/iterations) according to its own criteria (the optimization algorithm). At the end of the evolutive period of all populations it is necessary to identify the region of reference for each population (line 5). The metric chosen to define the region of reference is the centroid  $\vec{c}_i$  and represents the point in the space where there is a longest concentration of individuals of population  $i$ .

A key concept in the ECO system is the definition of habitats (line 6 in Algorithm 1). The ECO approach uses the single-link hierarchical clustering algorithm to set-up the habitats where each cluster represents a habitat. Hence, the habitats are defined probabilistically taking into account the distance information returned by the clustering algorithm [8].

Once defined the habitats, the next step in Algorithm 1 (line 7) is the definition of the communication topology for each habitat. This topology is probabilistically defined by using the distance information returned by a single linkage clustering algorithm [8] in such a way that the closer two population are from each other the higher is the chance of these two populations communicate. The opposite happens with farthest populations. After that, the mating ecological relationship between adjacent populations occurs (line 8). In line 9, the topology for interaction among habitats is randomly defined. This inter-habitats topology  $TH(t)$  is used for the migrations ecological relationship (line 10). The main loop continues until the ecological succession cycle reaches a maximum predefined value. For a detailed description refer to [8].

The ECO algorithm parameters are: number of populations ( $N-POP$ ) that will be co-evolved, initial population size ( $POP-SIZE$ ), number of cycles for ecological successions ( $ECO-STEP$ ), size of the evolutive period ( $EVO-STEP$ ) that represents number of function evaluations in each  $ECO-STEP$ , and tournament size ( $T-SIZE$ ) that is used to select individuals for intra-habitats communication.

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**Algorithm 1** Pseudo-code for ECO

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1: Let  $i = 1, \dots, N\text{-POP}$ ,  $j = 1, \dots, NH$  and  $t = 0$ ;  
2: Initialise each population  $Q_i^t$  with  $n_i$  random candidate solutions;  
3: while stop criteria not satisfied do {Ecological succession cycles}  
4:   Perform evolutive period for each population  $Q_i^t$ ;  
5:   Identify the region of reference  $\vec{c}_i$  for each population  $Q_i^t$ ;  
6:   Using the  $\vec{c}_i$  values, define the  $NH$  habitats;  
7:   For each habitat  $H_j^t$  define the communication topology  $CT_j^t$  between  
   populations  $Q_i^t$ ;  
8:   For each topology  $CT_j^t$ , perform interactions between populations  $Q_i^t$ ;  
9:   Define communication topology  $TH^t$  between  $H_j^t$  habitats;  
10:  Perform interactions between  $H_j^t$  habitats according to  $TH^t$ ;  
11:  Increase  $t$ ;  
12: end while
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### III. COMPUTATIONAL MODEL OF SYMBIOTIC RELATIONSHIPS

In order to increase the biological plausibility of the ECO approach, a new strategy for information exchange is added and it is inspired by the concept of symbiotic relationships. Symbiotic relationships are interactions that individuals perform with each other in the ecosystem resulting in benefits, damages or do not affecting those involved. In Nature, it is possible to observe several types of symbiotic relationships. The computational model is inspired by relationships of mutualism, altruism, slavery and competition.

A computational model inspired by symbiotic relationships should produce benefits and damages in populations of candidate solutions similarly to the biological phenomenon. Therefore, to model a symbiotic relationship it is necessary to define the concept of benefit and damages in the optimization context. The aim of an optimization strategy is to find promising regions in the search surface of a function  $f(\cdot)$ . In this way, the optimization process is benefited when the population of candidate solutions approaches a promising region. The opposite occurs when the optimization process is damaged. The population moves away from an promising region.

Symbiotic relationships are applied in intra-habitats communication phase of ECO algorithm by replacing the mating relationship that occurs in line 8 of Algorithm 1 by one of the possible symbiotic relationships. Intra-habitats communication topology ( $CT$ ) is used to select the pair of populations that will interact to each other. Each population  $Q_i$  in the computational ecosystem will interact with another population if it is part of a habitat with two or more populations.

The proposed strategy is based on the Pareto dominance criteria [14] and it is used to define which individuals of two interacting populations ( $I$  and  $J$ ) will exchange information. To find the non-dominated front the fitness values of individuals are used. Hence, the distribution of points on the Pareto front is directly influenced by the phenotypic diversity of involved populations. The individuals are grouped in pairs, one from each population, and their fitness values are used to create points on a Pareto front. According with the dominance criteria of each symbiotic relationship, described next, all non-dominated pairs in the Pareto front are identified and selected to exchange information. This differs from standard ECO in

which only one pair of solutions exchanges information within two interacting populations in the intra-habitat topology.

Benefits and damages of symbiotic relationships are related with the proposed model through the dominance criteria. For example, if the relationship benefits both populations, then their best individuals are selected. Otherwise, if the relationship damages the populations, then their worst individuals are selected. Figure 1 shows the Pareto front for each symbiotic relationship for a minimization problem. The  $x$ -axis and  $y$ -axis represent the fitness values of individuals in population  $I$  and  $J$ , respectively. In Figure 1, the signs (+) and (-) indicate benefit and damage to populations  $I$  and  $J$ , respectively.

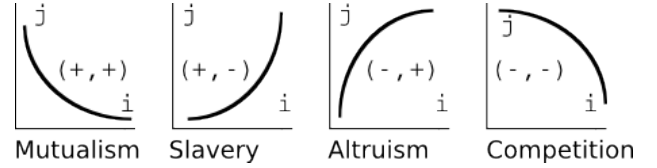


Fig. 1. Pareto front for each symbiotic relationship.

The mutualistic relationship occurs when two or more organisms interact and generate benefits for both. In this analogy, the selection of non-dominated individuals for mutualism is situated at the bottom-left distribution of pairs of individuals  $(I_x, J_y)$ . The dominance criteria is minimization for both populations. In this relationship populations are cooperatives, sharing best solutions. This is a relationship of the type (+, +) for ( $I, J$ ), respectively.

In the slavery relationship, the selection of non-dominated individuals is situated at the bottom-right distribution of pairs of individuals  $(I_x, J_y)$ . The dominance criteria is maximization for population  $I$  and minimization for population  $J$ . In this relationship population  $I$  takes advantage of another population with the intention of improving itself, regardless of population  $J$  and puts his worst individuals to exchange information with the best individuals of  $J$  in the non-dominated pairs. This is a relationship of the type (+, -) for ( $I, J$ ), respectively.

The altruism relationship occurs when an organism is harmed without the expectation of reciprocity or compensation for that action in order to benefit another organism. In this analogy, the selection of non-dominated individuals by altruism is situated at the top-left distribution of pairs of individuals  $(I_x, J_y)$ . The dominance criteria is minimization for population  $I$  and maximization for population  $J$ . In this relationship population,  $I$  does not have interest of improving itself and puts his best individuals to exchange information with the worst individuals of  $J$  in the non-dominated pairs. This is a relationship of the type (-, +) for ( $I, J$ ), respectively.

The competition relationship occurs when organisms compete against each other for environment resources and harm everyone involved. In this analogy, the selection of non-dominated individuals is situated at the top-right distribution of pairs of individuals  $(I_x, J_y)$ . The dominance criteria is maximization for both populations. In this relationship both

populations do not have interest to cooperate with each other. This is a relationship of the type  $(-, -)$  for  $(I, J)$ , respectively.

Algorithm 2 shows the general pseudo-code for the symbiotic relationships model. The algorithm starts getting two populations  $I$  and  $J$  (line 1) obtained from the intra-habitats communication topology ( $CT$ ). Next, pairs of individuals  $(I_x, J_y)$  with  $x$  and  $y = 1, \dots, POP\text{-}SIZE$ , are generated randomly (line 2). Pairs of individuals are generated randomly to avoid any kind of bias in the selection process. Randomly paired individuals are used to identify the Pareto front. The Pareto front (line 3) is the set of non-dominated pairs defined by the type of symbiotic relationship. The setting of a Pareto front determines how individuals will be selected and differentiates the symbiotic relationships. For each non-dominated pair the exchange of information is performed through uniform crossover (line 5). This operator generates two new individuals  $I_a$  and  $J_b$ . For each new individual it is verified whether it is better than its respective parent. If true, the new individual replaces its parent. Otherwise, the new individual is discarded (lines 6 to 11).

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#### Algorithm 2 Model of Symbiotic Relationships

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1: Input:  $POP_I, POP_J$ 
2: Generate  $POP\_SIZE$  pairs of individuals
3: Find non-dominated pairs {Symbiotic selection}
4: for Each non-dominated pair do
5:   Exchange of information between individuals
6:   if  $I_a$  is better than  $POP\_I_a$  then
7:     Replaces  $POP\_I_a$  by  $I_a$ 
8:   end if
9:   if  $J_b$  is better than  $POP\_J_b$  then
10:    Replaces  $POP\_J_b$  by  $J_b$ 
11:   end if
12: end for

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## IV. EXPERIMENTS AND RESULTS

All algorithms were developed using C++ language and experiments were run on an AMD Phenom II X4 (2.80GHz) with 4GB RAM, under Linux operating system. The experiments were conducted using thirty test functions extensively used in the literature for testing optimization methods [15]. Table I shows the domain and global minimum, respectively, for each test function.

For each test function, 30 independent runs were performed with randomly initialized populations. In all experiments the number of dimensions ( $d$ ) is equal to 200 and the parameters used were  $N\text{-}POP = 200$ ,  $ECO\text{-}STEP = 500$ ,  $EVO\text{-}STEP = 200$ ,  $POP\text{-}SIZE = 10$ ,  $T\text{-}SIZE = 5$  and a crossover rate of 50%. With this adjustment of parameters, the total number of functions evaluations was 100,000 evaluations for each population.

In all experiments, the Artificial Bee Colony Optimization (ABC) algorithm [13] was used to compound all ECO approaches in a homogeneous model, with parameter limit equal to 100, i.e. all populations use this algorithm with the same control parameters. All parameters were defined empirically [11].

Experiments were conducted using 6 different approaches. The first approach,  $ECO$ , refers to the canonical ecologically-inspired system explained in Section II. The  $ECO_s$ ,  $ECO_c$ ,  $ECO_a$  and  $ECO_m$  approaches refer to the ecological system using the symbiotic relationships of slavery, competition, altruism, and mutualism, respectively, as explained in Section III. Finally, the sixth approach,  $ABC$ , refers to the evolution of completely isolated populations of ABC algorithm, i.e., evolving without exchanging information.

Table II shows the results obtained. There are seven columns where the first column identifies the function and the remaining columns identify the following approaches:  $ABC$ ,  $ECO$ ,  $ECO_s$ ,  $ECO_c$ ,  $ECO_a$ , and  $ECO_m$ , respectively. The line of each function identifies the mean and standard deviation of the results obtained for each approach. To better analyze the results obtained, the pairwise comparison of *Mutualism* against other approaches was performed using the Wilcoxon-Rank Sum statistical test. The  $ECO_m$  approach was chosen as baseline approach due to verified best results. A significance level of 5% was employed. Best results that are statistically significant are highlighted in bold. From Table II, it is possible to observe that for 14 functions the  $ECO_m$  approach achieved better results.

The last row in Table II summarizes the performance of each approach when comparing with  $ECO_m$ . (B) refers to the number of functions in which the referred approach was statistically better than  $ECO_m$ , (S) shows the number of functions in which the referred approach obtained statistically the same results of  $ECO_m$ , and (W) shows the number of functions in which the referred approach obtained statistically worse results when compared to  $ECO_m$ . Figure 2 shows the performance of each approach compared to  $ECO_m$ .

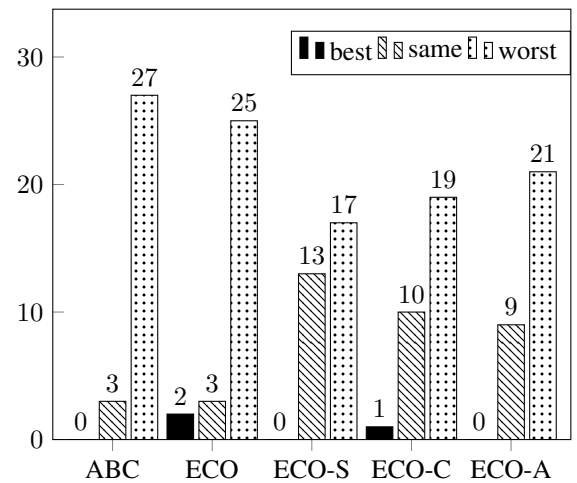


Fig. 2. Performance of each approach compared to  $ECO_m$

Analyzing  $ABC$  and  $ECO$  approaches we can observe that  $ECO$  obtained much better results than  $ABC$ . The use of co-evolutionary populations through ecological interactions enhance the performance gain [8].

Function	Domain	Min.	Function	Domain	Min.
1 Ackley	$[-32, 32]^d$	0	16 Schaffer F6	$[-100, 100]^d$	0
2 Egg Holder	$[-512, 512]^d$	$-915.61991n + 862.10466$	17 Schaffer F7	$[-100, 100]^d$	0
3 Generalized Holzman	$[-10, 10]^d$	0	18 Schwefel 2.22	$[-10, 10]^d$	0
4 Generalized Penalized Func. 1	$[-50, 50]^d$	1.57e-032	19 Shifted Ackley	$[-32, 32]^d$	-140
5 Generalized Penalized Func. 2	$[-50, 50]^d$	1.34e-032	20 Shifted Griewank	$[-600, 600]^d$	-180
6 Generalized Schwefels 2.26	$[-500, 500]^d$	-418.982887272433	21 Shifted Rastrigin	$[-5.12, 5.12]^d$	-330
7 Griewank	$[-600, 600]^d$	0	22 Shifted Rosenbrock	$[-100, 100]^d$	390
8 Levy	$[-10, 10]^d$	0	23 Shifted Schaffer	$[-100, 100]^d$	0
9 Michalewicz	$[0, \pi]^d$	$-0.99864n + 0.30271$	24 Shifted Schwefel Problem 2.21	$[-100, 100]^d$	-450
10 Molecular Potential Energy	$[0, 5]^d$	$-0.0411183034n$	25 Shifted Sphere	$[-100, 100]^d$	-450
11 Multimod	$[-10, 10]^d$	0	26 Shubert	$[-10, 10]^d$	-24.06
12 Powell	$[-4, 5]^d$	0	27 Sphere	$[-100, 100]^d$	0
13 Rana	$[-512, 512]^d$	$-511.70430n + 511.68714$	28 Step	$[-100, 100]^d$	0
14 Rastrigin	$[-5.12, 5.12]^d$	0	29 StretchedV	$[-10, 10]^d$	0
15 Rosenbrock	$[-30, 30]^d$	0	30 Zakharov	$[-5, 10]^d$	0

TABLE I  
TEST FUNCTIONS EMPLOYED IN THE EXPERIMENTS.

$f$	ABC	ECO	ECO <sub>s</sub>	ECO <sub>c</sub>	ECO <sub>a</sub>	ECO <sub>m</sub>
1	1.75e-3±2.95e-4	7.48e-07±5.01e-07	1.58e-08±1.82e-09	2.47e-07±2.18e-08	1.71e-08±1.60e-09	<b>7.41e-10±6.43e-11</b>
2	-122813±1230.56	-123822±3193.63	-132531±2903.31	-130473±2051.32	-131480±2519.38	-131987±2327
3	4.89e-16±2.07e-16	1.02e-24±8.13e-25	5.98e-30±2.71e-30	3.33e-26±2.24e-26	1.25e-29±6.99e-30	<b>1.61e-33±5.9e-34</b>
4	2.51e-09±7.27e-10	7.35e-16±6.89e-16	1.45e-18±5.22e-19	1.68e-16±3.89e-17	1.22e-18±3.35e-19	<b>2.69e-20±1.93e-21</b>
5	1.92e-06±4.788e-07	3.66e-4±1.97e-3	1.22e-07±6.32e-07	1.91e-12±2.63e-12	4.12e-15±3.1e-15	<b>1.11e-17±3.56e-18</b>
6	-395.84±1.42	-417.29±1.15	-418.98±1.13e-13	-418.98±1.13e-13	-418.98±1.13e-13	-418.98±1.13e-13
7	5.38e-09±1.40e-09	7.65e-15±2.28e-15	1.66e-17±2.79e-18	2.67e-15±4.1e-16	1.71e-17±2.85e-18	<b>1.20e-18±6.47e-20</b>
8	1.04e-08±2.74e-09	1.04e-14±2.85e-15	2.20e-17±3.0e-18	3.17e-15±6.21e-16	2.24e-17±3.15e-18	<b>5.84e-18±7.79e-21</b>
9	-188.34±0.74	-197.86±0.35	-199.42±0.04	<b>-199.57±0.007</b>	-199.4±0.049	-199.42±0.03
10	-7.1±0.13	-7.86±0.2	-8.22±3.55e-15	-8.22±3.55e-15	-8.22±3.55e-15	-8.22±3.55e-15
11	6,29e-4±6.11e-05	3.63e-08±3.9e-09	5.42e-10±6.12e-11	1.62e-08±9.9e-10	5.85e-10±5.34e-11	<b>1.52e-11±1.41e-12</b>
12	0.45±0.03	3.51±1.39	2.1±0.51	2.41±0.86	1.91±0.46	<b>1.32±0.35</b>
13	-390.91±3.57	-405.12±6.91	-428.07±7.49	-429.08±7.97	-424.70±4.84	-429.26±8.18
14	11.66±1.96	4.37±2.82	9.93e-15±3.91e-15	3.09e-12±6.27e-13	0.03±0.17	<b>1.3e-17±3.7e-18</b>
15	11.46±5.06	46.30±55.53	14.99±17.66	39.85±35.42	17.65±21.54	10.11±13.34
16	13.54±0.64	16.77±3.33	12.24±1.65	11.81±1.45	13.24±2.23	12.12±1.38
17	26.45±0	<b>11.37±0.81</b>	23.09±0.78	26.19±0.46	23.43±0.86	21.17±0.69
18	3,61e-4±2.31e-15	7.16e-08±8.04e-09	1.07e-09±1.11e-10	3.40e-08±2.15e-09	1.19e-09±1.07e-10	<b>2.94e-11±2.70647e-12</b>
19	-139.98±2,97e-3	-139.99±0.04	-140±0	-140±0	-140±0	-140±0
20	-180±0	-180±0	-180±0	-180±0	-180±0	-180±0
21	-319.32±1.13	-321.43±3.43	-330±0	-330±0	-329.99±0.002	-330±0
22	479.20±26.49	821.93±1813.03	478.76±37.28	508.87±43.77	466.64±30.79	467.39±30.8
23	59.3±4.12	169.07±69.19	58.33±35.87	63.48±37.08	50.87±26.72	<b>25.25±18.69</b>
24	-303±2.83	-357.22±3.87	-358.07±5.37	-355.99±4.87	-356.02±3.54	-354.26±3.25
25	-450±0	-450±0	-450±0	-450±0	-450±0	-450±0
26	-23.85±0.03	-24.04±0.02	-24.06±0	-24.06±0	-24.06±1.79e-05	-24.06±0
27	3.76e-07±8.17e-8	3.97e-13±1.17e-13	1.04e-15±2.19e-16	2.61e-13±5.49e-14	1.02e-15±2.19e-16	<b>3.42e-18±6.18e-19</b>
28	3.47±8.14e-08	3.63e-13±8.49e-14	1.02e-15±2.32e-16	2.45e-13±4.29e-14	1.05e-15±2.03e-16	<b>3.36e-18±5.26e-19</b>
29	16.497±1.06	37.98±18.11	15.10±8.57	12.44±8.22	11.11±7.53	<b>5.19±5.35</b>
30	2601.62±62.8	<b>392.39±165.96</b>	1475.12±139.86	1377.34±145.78	1350.36±142.61	1227.21±159.28
<b>B/S/W</b>	<b>0/3/27</b>	<b>2/3/25</b>	<b>0/13/17</b>	<b>1/10/19</b>	<b>0/9/21</b>	

TABLE II  
RESULTS OBTAINED FOR TEST FUNCTIONS.

The results also show that all symbiotic approaches obtained better results than the canonical *ECO* approach in almost all functions. *ECO<sub>m</sub>* achieved best results in 25 test functions when compared to *ECO* approach. This indicates that the use of symbiotic relationships favors the balance of exploration and exploitation during the search process. This advantage can also be observed when comparing *ECO<sub>m</sub>* approach with other

symbiotic approaches.

The good performance of *ECO<sub>m</sub>* can be explained by the choice of good individuals to exchange information through the Pareto front. The Pareto front for *ECO<sub>m</sub>* is proportional to the objective being optimized: minimization. Also, as well as in nature, mutualistic relationships benefit both populations, and in the optimization context, contribute to a better explo-

ration of the solution space. This indicates that the model has better ability to identify promising areas in the search space.

## V. CONCLUSION

Biological phenomena can influence different aspects of bio-inspired systems. Many of these systems are inspired by some biological characteristics. In nature, systems are interconnected forming biological ecosystems. In this way, the inclusion of new features to provide greater biological plausibility in optimization algorithms may increase their efficiency and robustness to handle complex problems.

This work presents a computational model for information exchange between populations inspired by symbiotic relationships of organisms. With the analogy based on symbiotic relationships, each population interacts with another resulting in benefits or damages to them.

Also, the symbiotic model employed for information exchange is parameter free and uses different strategies to select individuals. Pairs of individuals are selected using the non-domination criteria of Pareto. Depending on the criterion of selection, selected individuals have low or high quality solutions to exchange information. Also, this strategy suppressed the use of the parameter *T-SIZE* in standard ECO.

The proposed model was applied in the ecologically-inspired algorithm adding a new biologically plausible mechanism. The Artificial Bee Colony Optimization was used to compose the ecological framework. Thirty continuous test functions with a high number of dimensions ( $d = 200$ ) were employed in the experiments. Results suggest that the proposed model for information exchange in an ecosystemic context favors the balance between exploration and exploitation leading to better results. The results obtained in this test set support the hypothesis that a system with more biologically plausible features is able to increase its efficiency and robustness when facing a high number of dimensions in single objective unconstrained continuous benchmark functions. In our experiments, the mutualistic approach achieved best results.

There are several research directions for future developments. Some of them are to analyse the computational complexity; to implement diversity maintenance strategies; use an automatic mechanism to set the symbiotic selection; calculate and use phenotypic and genotypic diversity informations to adjust the system parameters; explore other symbiotic relationships in the system; and apply the model in other continuous and discrete optimization problems.

Finally, as any computational biological abstraction, the symbiotic model for information exchange presented in this work does not accomplish the whole complexity that occurs in a real ecosystem but shows some potential for application in optimization problems.

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